9/10/19

MIS3: Life Tables & Common Mortality Assumptions

Life Table Notation

$$l_x = \text{"expected" \# of lives at age } x$$

$$l_0 = \text{\# of newborns (arbitrary)}$$

$$_nP_x = \frac{l_{x+n}}{l_x}$$

$$_nq_x = 1 - {_nP_x} = 1 - \frac{l_{x+n}}{l_x} = \frac{l_x - l_{x+n}}{l_x} = \frac{_nd_x}{l_x}$$

($_nd_x = l_x - l_{x+n}$ = # of x-year olds who die in the next n years)

Example: $${_{10}P_{20}} = \frac{l_{30}}{l_{20}} \overset{\text{SULT}}{=} \frac{99727.3}{100000} = 0.997273$$

$$_{k|n}q_x = \frac{_nd_{x+k}}{l_x} = \frac{l_{x+k} - l_{x+k+n}}{l_x}$$

Remark: $$= \frac{l_{x+k}}{l_x} \cdot \frac{l_{x+k} - l_{x+k+n}}{l_{x+k}} = {_kP_x} \cdot {_nq_{x+k}}$$

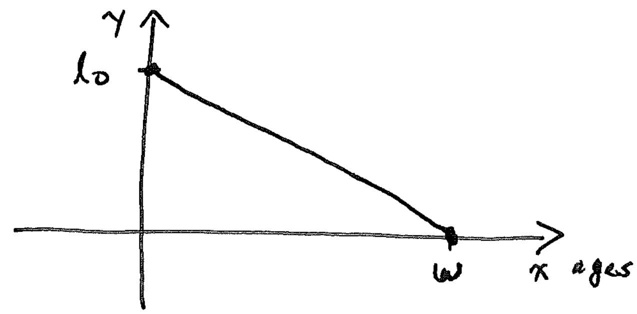
Common Mortality Models

I) De Moivre's Law DML(ω)

↳ pronounced "De Marve"

↳ omega "terminal age"

Graph of $y = l_x$



<u>Remarks</u>: 1) X = rvr age at death of a newborn

<u>Note</u>: $X = T_0$

2) For DML(ω) mortality

$$X \sim U(0, \omega)$$

3) $T_x = X - x \mid X > x$

$$\Longrightarrow T_x \sim U(0, \omega - x)$$

all DML(ω) facts

$\therefore$ pdf: $f_x(t) = \frac{1}{\omega - x}$ (constant wrt "t")

cdf: $F_x(t) = {}_tq_x = \frac{t}{\omega - x}$ $(= t \cdot q_x)$

sf: $S_x(t) = {}_tp_x = \frac{\omega - x - t}{\omega - x}$

fom: $\mu_{x+t} = \frac{1}{\omega - x - t}$

$\mathring{e}_x = \frac{\omega - x}{2}$ $Var(T_x) = \frac{(\omega - x)^2}{12}$ $e_x = \mathring{e}_x - \frac{1}{2}$

Test 2 Review

m151

$$\mathring{e}_x = \int_0^\infty {}_tP_x \, dt$$

$$e_x = \sum_{k=1}^{\infty} {}_kP_x$$

Note: $\int a^t \, dt = \dfrac{a^t}{\ln(a)} + c$

---

m152

$$\int_0^n f_x(t)\,dt = \int_0^n {}_tP_x \cdot \mu_{x+t}\,dt = {}_nq_x$$

$$\int_0^n \mu_{x+t}\,dt \rightsquigarrow {}_nP_x = e^{-\int_0^n \mu_{x+t}\,dt}$$

$$\int_x^{x+n} \mu_r\,dr \rightsquigarrow {}_nP_x = e^{-\int_x^{x+n} \mu_r\,dr}$$